Lösung zu S.86/21:

$\overrightarrow{A'} = \vec{b} + \frac{2}{3}\overrightarrow{BC} = \vec{b} + \frac{2}{3}(\vec{c} - \vec{b}) = \frac{1}{3}\vec{b} + \frac{2}{3}\vec{c};$

$\overrightarrow{B'} = \vec{c} + \frac{1}{4}\overrightarrow{CA} = \vec{c} + \frac{1}{4}(\vec{a} - \vec{c}) = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{c};$

$\overrightarrow{C'} = \vec{a} + \frac{1}{3}\overrightarrow{AB} = \vec{a} + \frac{1}{3}(\vec{b} - \vec{a}) = \frac{2}{3}\vec{a} + \frac{1}{3}\vec{b};$

$\vec{X} = \vec{a} + \overrightarrow{AX} = \vec{a} + \lambda\overrightarrow{AA'} = \vec{a} + \lambda(\overrightarrow{A'} - \vec{a}) = \vec{a} + \lambda\left(\frac{1}{3}\vec{b} + \frac{2}{3}\vec{c} - \vec{a}\right)$

$\vec{Y} = \vec{a} + \overrightarrow{AY} = \vec{a} + r\overrightarrow{AA'} = \vec{a} + r(\overrightarrow{A'} - \vec{a}) = \vec{a} + r\left(\frac{1}{3}\vec{b} + \frac{2}{3}\vec{c} - \vec{a}\right)$

$\overrightarrow{AX} + \overrightarrow{XB'} + \overrightarrow{B'A} = \vec{O}$

$$\overrightarrow{AX} = \lambda\overrightarrow{AA'} = \lambda(-\vec{a} + \vec{b} + \overrightarrow{BA'}) = \lambda\left(-\vec{a} + \vec{b} + \frac{2}{3}\overrightarrow{BC}\right) = \lambda\left(-\vec{a} + \vec{b} + \frac{2}{3}(-\vec{b} + \vec{c})\right)$$

$$= -\lambda\vec{a} + \left(1 - \frac{2}{3}\right)\lambda\vec{b} + \frac{2}{3}\lambda\vec{c} = -\lambda\vec{a} + \frac{1}{3}\lambda\vec{b} + \frac{2}{3}\lambda\vec{c}$$

oder schneller:

$$\overrightarrow{AX} = \lambda\overrightarrow{AA'} = \lambda(\overrightarrow{A'} - \vec{a}) = \lambda\left(\frac{1}{3}\vec{b} + \frac{2}{3}\vec{c} - \vec{a}\right)$$

$$\overrightarrow{XB'} = s\overrightarrow{C'B'} = s\left(\left(\frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}\right) - \left(\frac{2}{3}\vec{a} + \frac{1}{3}\vec{b}\right)\right) = s\left(-\frac{5}{12}\vec{a} - \frac{1}{3}\vec{b} + \frac{3}{4}\vec{c}\right)$$

$$\overrightarrow{B'A} = \vec{A} - \vec{B'} = \vec{a} - \left(\frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}\right) = \frac{3}{4}\vec{a} - \frac{3}{4}\vec{c};$$

$\overrightarrow{AX} + \overrightarrow{XB'} + \overrightarrow{B'A} = \vec{O}$

$\left(-\lambda\vec{a} + \frac{1}{3}\lambda\vec{b} + \frac{2}{3}\lambda\vec{c}\right) + \left(s\left(-\frac{5}{12}\vec{a} - \frac{1}{3}\vec{b} + \frac{3}{4}\vec{c}\right)\right) + \left(\frac{3}{4}\vec{a} - \frac{3}{4}\vec{c}\right) = \vec{0}$

$\left(-\lambda - \frac{5s}{12} + \frac{3}{4}\right)\vec{a} + \left(\frac{\lambda}{3} - \frac{s}{3}\right)\vec{b} + \left(\frac{2\lambda}{3} + \frac{3s}{4} - \frac{3}{4}\right)\vec{c} = \vec{0}$

$$\begin{array}{llll} \text{I} & -\lambda & -\frac{5}{12}s & = -\frac{3}{4} \\ \text{II} & \lambda & -s & = 0 \\ \text{III} & \frac{2}{3}\lambda & +\frac{3}{4}s & = \frac{3}{4} \end{array}$$

Aus (II) folgt: $\lambda = s$

In (I): $-s - \frac{5}{12}s = -\frac{3}{4} \Rightarrow \frac{17}{12}s = \frac{3}{4} \Rightarrow s = \frac{3 \cdot 12}{4 \cdot 17} = \frac{9}{17};$

In (III): $\frac{2}{3}s + \frac{3}{4}s = \frac{8+9}{12}s = \frac{17}{12}s = \frac{3}{4} \Rightarrow s = \frac{3 \cdot 12}{4 \cdot 17} = \frac{9}{17};$ OK!

d.h. $\overrightarrow{AX} = \frac{9}{17}\overrightarrow{AA'} \Rightarrow TV(AXA') = \frac{9}{8};$

Analog lässt sich zeigen: $TV(AYA') = \frac{3}{2};$