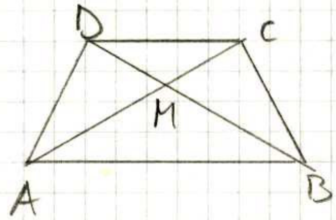
Lösung GK-Abi 2001 V

Musterlösung, Abi 2001, Analytische Geometrie, V

Geg.: $A(-2|8|0)$; $B(0|0|-2)$; $C(1|2|0)$; $D(0|6|1)$

5

1 a)



$\vec{AB} = \begin{pmatrix} 2 \\ -8 \\ -2 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ 1

$\vec{DC} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$

$\vec{AD} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

$\vec{AB} = 2 \cdot \vec{DC} \Rightarrow AB \parallel DC$ 1,5

$\left.\begin{matrix} |\vec{BC}| = \sqrt{1+4+4} = \sqrt{9} = 3 \\ |\vec{AD}| = 3 \end{matrix}\right\} \Rightarrow \overline{BC} = \overline{AD}$ 1,5

$\vec{BC}$ und $\vec{AB}$ sind linear unabhängig $\Rightarrow BC \nparallel AD$ 1

b) $g: \vec{x} = \vec{A} + r \cdot \vec{AC} = \begin{pmatrix} -2 \\ 8 \\ 0 \end{pmatrix} + r \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix}$ 0,5

$h: \vec{x} = \vec{B} + s \cdot \vec{BD} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + s \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix}$ 0,5

$\{M\} = h \cap g$: 1 $\left\{\begin{matrix} \text{(I)} & -2 + 3r = 0 + 0s & \Rightarrow r = \frac{2}{3} & 1 \\ \text{(II)} & 8 - 6r = 0 + 6s & & \\ \text{(III)} & 0 + 0r = -2 + 3s & \Rightarrow s = \frac{2}{3} & 1 \end{matrix}\right.$

r und s in (II): $8 - 6 \cdot \frac{2}{3} = 6 \cdot \frac{2}{3}$ ✓ nicht nötig!

$\vec{M} = \vec{B} + \frac{2}{3}\vec{BD} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ 1

$M(0|4|0)$ Abzug ggf. 0,25

1c) Hilfsebene H durch D mit Normalenvektor $\overrightarrow{AB}$

1 H: $\overrightarrow{AB} \circ (\vec{X} - \vec{D}) = 0$

$$\begin{pmatrix}2\\-8\\-2\end{pmatrix} \circ \left(\vec{X} - \begin{pmatrix}0\\6\\1\end{pmatrix}\right) = 0$$

$$2x_1 - 8x_2 - 2x_3 - (-8\cdot 6 - 2\cdot 1) = 0$$

1 $$2x_1 - 8x_2 - 2x_3 + 50 = 0$$

Gerade AB: $\vec{X} = \vec{A} + t\,\overrightarrow{AB} = \begin{pmatrix}-2\\8\\0\end{pmatrix} + t\begin{pmatrix}2\\-8\\-2\end{pmatrix}$

$\{F\} = H \cap AB$:

1 $$2(-2+2t) - 8(8-8t) - 2(-2t) + 50 = 0$$

$$-4 + 4t - 64 + 64t + 4t + 50 = 0$$

$$72t = +18$$

$$t = +\frac{1}{4}$$

1 $$\vec{F} = \vec{A} + \frac{1}{4}\overrightarrow{AB} = \begin{pmatrix}-2\\8\\0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}2\\-8\\-2\end{pmatrix} = \begin{pmatrix}-1{,}5\\6\\-0{,}5\end{pmatrix}$$

$$\overrightarrow{DF} = \begin{pmatrix}-1{,}5\\0\\-1{,}5\end{pmatrix}$$

1 $$d = |\overrightarrow{DF}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2};$$

d) $$A_{ABCD} = \frac{\overline{AB} + \overline{DC}}{2}\cdot d = \frac{\left|\begin{pmatrix}2\\-8\\-2\end{pmatrix}\right| + \left|\begin{pmatrix}1\\-4\\-1\end{pmatrix}\right|}{2}\cdot\frac{3}{2}\sqrt{2}$$

1 0,5

0,5 0,5

$$= \frac{\sqrt{4+64+4} + \sqrt{1+16+1}}{2}\cdot\frac{3}{2}\sqrt{2}$$

$$= \frac{\sqrt{72} + \sqrt{18}}{2}\cdot\frac{3}{2}\sqrt{2} = \frac{6\sqrt{2} + 3\sqrt{2}}{2}\cdot\frac{3}{2}\sqrt{2}$$

1,5 $$= \frac{9\sqrt{2}}{2}\cdot\frac{3}{2}\sqrt{2} = \frac{27}{2};$$

e) (I) $\vec{n} \circ \vec{AB} = 0 \Leftrightarrow \vec{n} \circ \begin{pmatrix} 2 \\ -8 \\ -2 \end{pmatrix} = 0$

(II) $\vec{n} \circ \vec{AD} = 0 \Leftrightarrow \vec{n} \circ \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 0$

(I) $2n_1 - 8n_2 - 2n_3 = 0$

(II) $2n_1 - 2n_2 + n_3 = 0$

(I') $2n_1 - 8n_2 = 2n_3$

(II') $2n_1 - 2n_2 = -n_3$

(I') − (II'): $-6n_2 = 3n_3 \Rightarrow n_2 = -\frac{1}{2} n_3$

n_2 in (II'): $2n_1 - 2\left(-\frac{1}{2} n_3\right) = -n_3$

$$2n_1 = -2n_3$$

$$n_1 = -n_3$$

$\Rightarrow \vec{n} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ 1 \end{pmatrix} \cdot r$ z.B. $\vec{n} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ 3

E: $\vec{n} \circ (\vec{X} - \vec{A}) = 0$ 1

$-2x_1 - x_2 + 2x_3 - (4-8) = 0$

$-2x_1 - x_2 + 2x_3 + 4 = 0$ 1

2a) $\vec{n}' = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ zeigt vom Ursprung weg 1

$$\vec{n}^{\circ} = \frac{1}{\sqrt{4+1+4}} \cdot \vec{n}' = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \quad 1$$

$$\vec{S} = \vec{M} + 15 \cdot \vec{n}^{\circ} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + 15 \cdot \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 10 \\ 5 \\ -10 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \\ -10 \end{pmatrix} \quad 1$$

$S(10|9|-10)$ −0,25

b) $$|\vec{MT}| = \left|\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}\right| = \sqrt{36+9+36} = 9 \quad 1$$

$$|\vec{TS}| = \left|\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}\right| = \sqrt{16+4+16} = 6 \quad 1$$

$$\frac{|\vec{MT}|}{|\vec{TS}|} = \frac{9}{6} = \frac{3}{2}; \; 1 \qquad \vec{MT} \cdot \frac{2}{3} = \vec{TS} \quad 1$$

c) $$F: \vec{n}' \circ (\vec{X} - \vec{T}) = 0$$

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \circ \left(\vec{X} - \begin{pmatrix} 6 \\ 7 \\ -6 \end{pmatrix}\right) = 0$$

1 0,5

$$2x_1 + x_2 - 2x_3 - (12 + 7 + 12) = 0$$

$$2x_1 + x_2 - 2x_3 - 31 = 0 \quad 1$$

d) $$\frac{V_{\text{Ergänzungpp.}}}{V_{\text{Pyramide ABCDS}}} = \left(\frac{2}{2+3}\right)^3 = \frac{8}{125} \approx 6{,}4\,\%;$$ 6

$$V_{\text{Ergänzungpp.}} = \frac{1}{3}A^* \cdot |\vec{TS}| = \frac{1}{3} m^* \cdot h^* \cdot |\vec{TS}|;$$

$$V_{\text{Pyramide}} = \frac{1}{3}A \cdot |\vec{MS}| = \frac{1}{3} m\, h \cdot |\vec{MS}|$$

mit $$\frac{m^*}{m} = \frac{|\vec{TS}|}{|\vec{MS}|} = \frac{h^*}{h}$$