Abitur 2001 GK

II

$$f_k(x) = \frac{2x-k}{(x+k)^2} \qquad k>0$$

1a $x+k=0$; $x=-k$ $\qquad D_k = \mathbb{R}\setminus\{-k\}$

6 $\lim\limits_{x\to\infty} \frac{2x-k}{(x+k)^2} = 0 = \lim\limits_{x\to-\infty} \frac{2x-k}{(x+k)^2}$

$\lim\limits_{x \overset{>}{\to} -k} \frac{\overbrace{2x-k}^{\to -3k}}{\underbrace{(x+k)^2}_{\to +0}} = -\infty \qquad \lim\limits_{x \overset{<}{\to} -k} \frac{\overbrace{2x-k}^{\to -3k}}{\underbrace{(x+k)^2}_{\to +0}} = -\infty$

senkr. Asymptote $x=-k$ hor. As. $y=0$

2 b) N: $2x-k=0$; $x=\frac{k}{2}$ $\quad N(\frac{k}{2}|0)$

$x=0$: $f(0) = \frac{-k}{k^2} = -\frac{1}{k}$ $\quad S(0|-\frac{1}{k})$

2a) $f_k'(x) = \frac{(x+k)^2\cdot(2) - (2x-k)\cdot 2(x+k)}{(x+k)^4} =$

$= \frac{(x+k)\cdot[2x+2k-4x+2k]}{(x+k)^4} =$

$= \frac{(x+k)\cdot(-2x+4k)}{(x+k)^4} = \frac{-2x+4k}{(x+k)^3}$

9 $f_k'(x)=0 \quad -2x+4k=0$; $2x=4k$; $x=2k$

$f_k''(x) = \frac{(x+k)^3\cdot(-2) - (-2x+4k)\cdot 3(x+k)^2}{(x+k)^6}$

$f_k''(2k) = \frac{(3k)^3(-2) - 0}{(3k)^6} = \frac{3^3 k^3\cdot(-2}{3^6 k^6}$

$= \frac{-2}{27k^3} < 0 \Rightarrow H(2k|\frac{1}{3k})$

$f(2k) = \frac{4k-k}{(3k)^2} = \frac{3k}{9k^2} = \frac{1}{3k}$

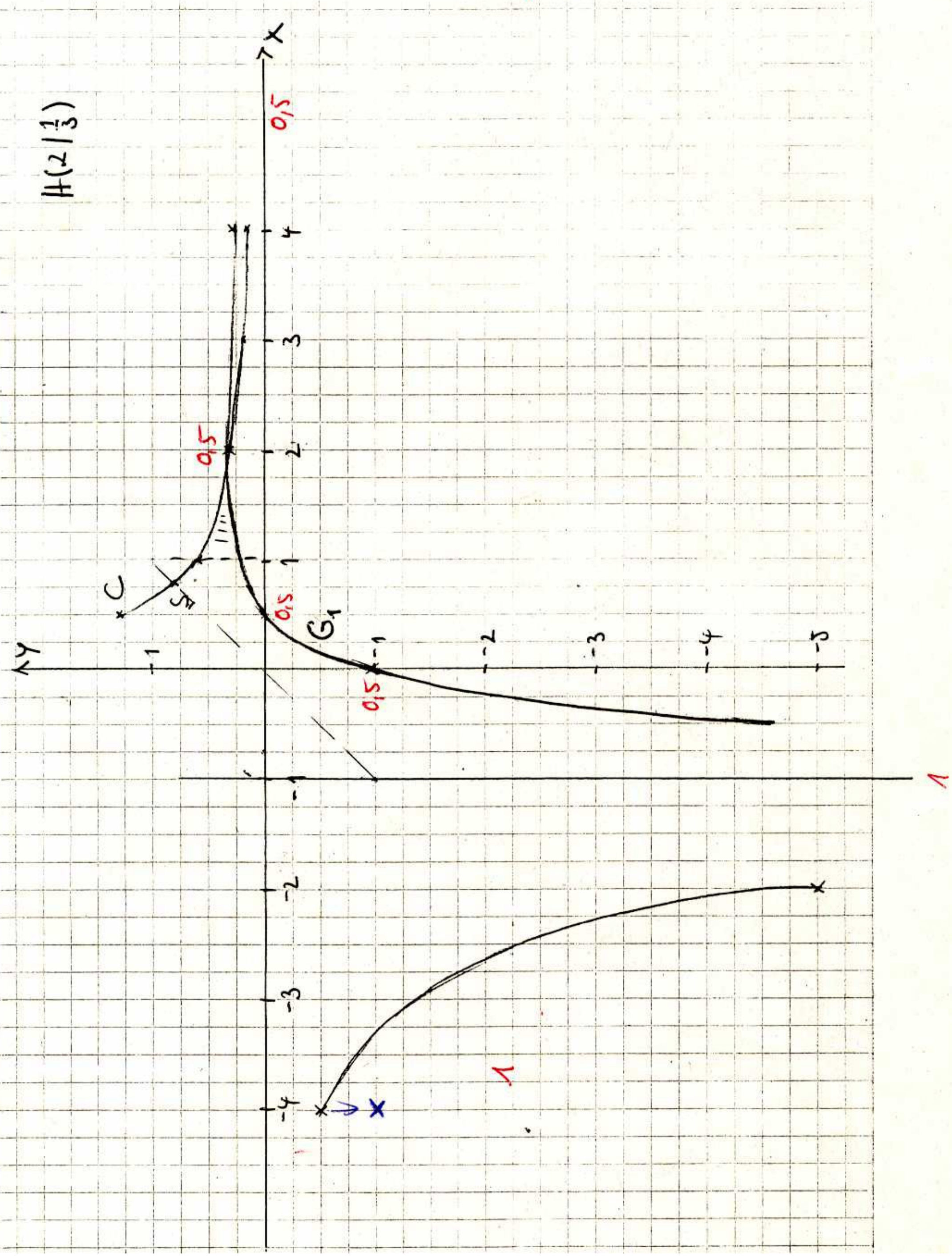
b) $f_1(-4) = \frac{-8-1}{(-3)^2} = -1$ $\qquad f_1(x) = \frac{2x-1}{(x+1)^2}$

$f_1(-2) = \frac{-4-1}{(-1)^2} = -5$ $\qquad f_1(4) = \frac{8-1}{(5)^2} = \frac{7}{25} = 0{,}28$

$f_1(6) = \frac{11}{7^2} = 0{,}2245$ $\qquad f_1(-6) = \frac{-13}{25} = -0{,}52$

2 P

−0,5 pro Fehler



1c) $x = 2k \quad y = \frac{1}{3k}$

2 $\quad k = \frac{x}{2} \Rightarrow y = \frac{1}{3 \cdot \frac{x}{2}} \qquad y = \frac{2}{3x}$

2 $\quad$ Schnittpkt: $x = \frac{2}{3x}$; $3x^2 = 2$; $x^2 = \frac{2}{3}$ $\quad x = \sqrt{\frac{2}{3}}$ $\left(x = \underbrace{-\sqrt{\frac{2}{3}}}\right)$

−0,5 falls stehen gelassen

2 $\quad S_1\left(\sqrt{\frac{2}{3}} \,\middle|\, \sqrt{\frac{2}{3}}\right)$

Kurve C:

x	0,5	1	2	3	4	0,8
y	1,3	0,6	0,3	0,2	0,17	0,8

2

3. a) $F'(x) = 2 \cdot \frac{1}{x+1} + \frac{(x+1)\cdot(-2) - (1-2x)}{(x+1)^2} =$ 2

1,5 $\quad$ 0,5

4 $\quad = \frac{2(x+1) - 2x - 2 - 1 + 2x}{(x+1)^2} =$

1

$= \frac{2x + 2 - 2x - 2 - 1 + 2x}{(x+1)^2} = \frac{2x-1}{(x+1)^2} = f_1(x)$

b) 1 $\int_1^2 \left(\frac{2}{3x} - f_1(x)\right) dx = \left[\frac{2}{3}\ln x - 2\ln(x+1) - \frac{1-2x}{x+1}\right]_1^2 =$

2 $\quad$ 1

$= \frac{2}{3}\ln 2 - 2\ln 3 - \frac{-3}{3} - \left(0 - 2\ln 2 - \frac{-1}{2}\right) =$

2 $\quad = \frac{2}{3}\ln 2 - 2\ln 3 + 1 + 2\ln 2 - \frac{1}{2} = \frac{8}{3}\ln 2 - 2\ln 3 + \frac{1}{2} \approx$

$\approx 0{,}15$

1