

Berechne:

$$1. (x^{-5}+y^{-5})(x^{-5}-y^{-5})=x^{-10}-y^{-10}$$

$$2. (a+a^{-1})^2-(a-a^{-1})^2=(a^2+2aa^{-1}+a^{-2})-(a^2-2aa^{-1}+a^{-2})=4$$

$$3. (x^{-4}+1)(x^{-4}-1)-(x^4-x^{-4})^2=(x^{-8}-1)-(x^8-2x^4x^{-4}+x^{-8})=1-x^8$$

$$4. (a^0-a^{-1})^{-1}-(a^0+a^{-1})^{-1}=\left(1-\frac{1}{a}\right)^{-1}-\left(1+\frac{1}{a}\right)^{-1}=\left(\frac{a-1}{a}\right)^{-1}-\left(\frac{a+1}{a}\right)^{-1}=\frac{a}{a-1}-\frac{a}{a+1}=\frac{a(a+1)-a(a-1)}{(a-1)(a+1)}=\frac{2a}{a^2-1}$$

Berechne:

$$\begin{aligned} 1. \quad \left(\frac{8c^{-5}}{9a^{-3}b^9}\right)^{-3} \cdot \left(\frac{3a^{-2}c^3}{4b^{-5}}\right)^{-5} &= \left(\frac{8^{-3}c^{15}}{9^{-3}a^9b^{-27}}\right) \cdot \left(\frac{3^{-5}a^{10}c^{-15}}{4^{-5}b^{25}}\right) = \frac{2^{-9}c^{15-15}3^{-5}a^{10-9}}{3^{-6}b^{-27+25}2^{-10}} \\ &= 2^{-9-(-10)} \cdot 3^{-5-(-6)} ab^2 = 2 \cdot 3ab^2 = 6ab^2 \end{aligned}$$
$$\begin{aligned} 2. \quad \left(\frac{1}{4}\right)^{-3} \cdot \left(\frac{a^{-2}b^{-1}}{a^3b^2}\right)^3 : \left(\frac{a^7}{8b^{-4}}\right)^{-2} &= 4^3 \cdot (a^{-5}b^{-3})^3 : \left(\frac{8b^{-4}}{a^7}\right)^2 = (4^3 a^{-15} b^{-9}) : \left(\frac{8^2 b^{-8}}{a^{14}}\right) \\ &= 2^6 a^{-15} b^{-9} \cdot \left(\frac{a^{14}}{2^6 b^{-8}}\right) = a^{-1} b^{-1} = \frac{1}{ab} \end{aligned}$$

Berechne (natürlich OHNE Taschenrechner!):

$$\begin{aligned} 1. \quad (-3x^{-4})^{-3} \cdot \left(\frac{2}{3x^4}\right)^{-2} + \frac{4}{3} \cdot \left[\frac{2}{(-x)^4}\right]^{-5} &= -3^{-3} x^{12} \cdot \left(\frac{3x^4}{2}\right)^2 + \frac{4}{3} \left(\frac{x^4}{2}\right)^5 = -3^{-3} x^{12} \cdot \left(\frac{3^2 x^8}{2^2}\right) + \frac{4}{3} \left(\frac{x^{20}}{2^5}\right) \\ &= -\frac{3^{-1}}{2^2} \cdot x^{20} + \frac{2^2}{3 \cdot 2^5} x^{20} = -\frac{1}{12} x^{20} + \frac{1}{24} x^{20} = -\frac{1}{24} x^{20} \end{aligned}$$

$$\begin{aligned} 2. \quad \left(\frac{1}{0,25}\right)^{-2} \cdot \left\{2 \cdot \left[\frac{1}{(-c)^5}\right]^{-4} + \frac{2}{5} \cdot (-c^4)^5\right\} &= 4^{-2} \cdot \left\{2 \cdot [(-c)^5]^4 + \frac{2}{5} \cdot (-c^{20})\right\} = 2^{-4} \cdot \left\{2 \cdot c^{20} - \frac{2}{5} c^{20}\right\} \\ &= 2^{-4} \cdot \frac{8}{5} c^{20} = 2^{-4} \cdot \frac{2^3}{5} c^{20} = \frac{1}{10} c^{20} \end{aligned}$$