VI Abitur 2004

1a) $\overrightarrow{AB} = \vec{B} - \vec{A} = \begin{pmatrix}-10\\4\\0\end{pmatrix}$; $\overrightarrow{BS} = \vec{S} - \vec{B} = \begin{pmatrix}0\\-4\\6\end{pmatrix}$;

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{BS} = \begin{pmatrix}-10\\4\\0\end{pmatrix} \times \begin{pmatrix}0\\-4\\6\end{pmatrix} = \begin{pmatrix}24\\60\\+40\end{pmatrix} = 4 \cdot \underbrace{\begin{pmatrix}6\\15\\+10\end{pmatrix}}_{\vec{n}^*}$$

$$F: \ \underset{4}{\vec{n}} \circ (\vec{X} - \vec{A}) = 0$$

$$\begin{pmatrix}6\\15\\10\end{pmatrix} \circ \left[\vec{X} - \begin{pmatrix}10\\0\\0\end{pmatrix}\right] = 0$$

$$6x_1 + 15x_2 + 10x_3 - 60 = 0$$

b) Normalenvektor von F : $\vec{n}^* = \begin{pmatrix}6\\15\\10\end{pmatrix}$

" " x_1x_2-Ebene : x_3-Richtung also $\begin{pmatrix}0\\0\\1\end{pmatrix}$

$$\cos\varphi = \frac{\left|\begin{pmatrix}6\\15\\10\end{pmatrix} \circ \begin{pmatrix}0\\0\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}6\\15\\10\end{pmatrix}\right| \cdot \left|\begin{pmatrix}0\\0\\1\end{pmatrix}\right|} = \frac{10}{\sqrt{36+225+100} \cdot 1} = \frac{10}{\sqrt{361}}$$

$$= \frac{10}{19}$$

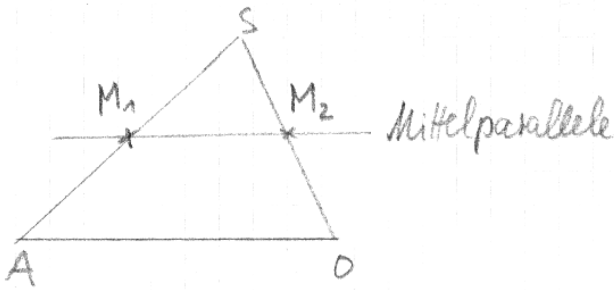
$$\Rightarrow \varphi \approx 58{,}243^\circ$$

c) Normalenvektor von E_2 : $\vec{n}_2 = \begin{pmatrix}0\\3\\2\end{pmatrix}$

$$\overrightarrow{BS} \circ \vec{n}_2 = \begin{pmatrix}0\\-4\\6\end{pmatrix} \circ \begin{pmatrix}0\\3\\2\end{pmatrix} = -12 + 12 = 0$$

$$\Rightarrow \overrightarrow{BS} \perp \vec{n}_2 \Rightarrow BS \parallel E_2$$

1d)



$$\vec{M_1} = \frac{\vec{A}+\vec{S}}{2} = \frac{1}{2}\left[\begin{pmatrix}10\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\6\end{pmatrix}\right] = \begin{pmatrix}5\\0\\3\end{pmatrix}$$

$$\vec{M_2} = \frac{\vec{O}+\vec{S}}{2} = \frac{1}{2}\begin{pmatrix}0\\0\\6\end{pmatrix} = \begin{pmatrix}0\\0\\3\end{pmatrix}$$

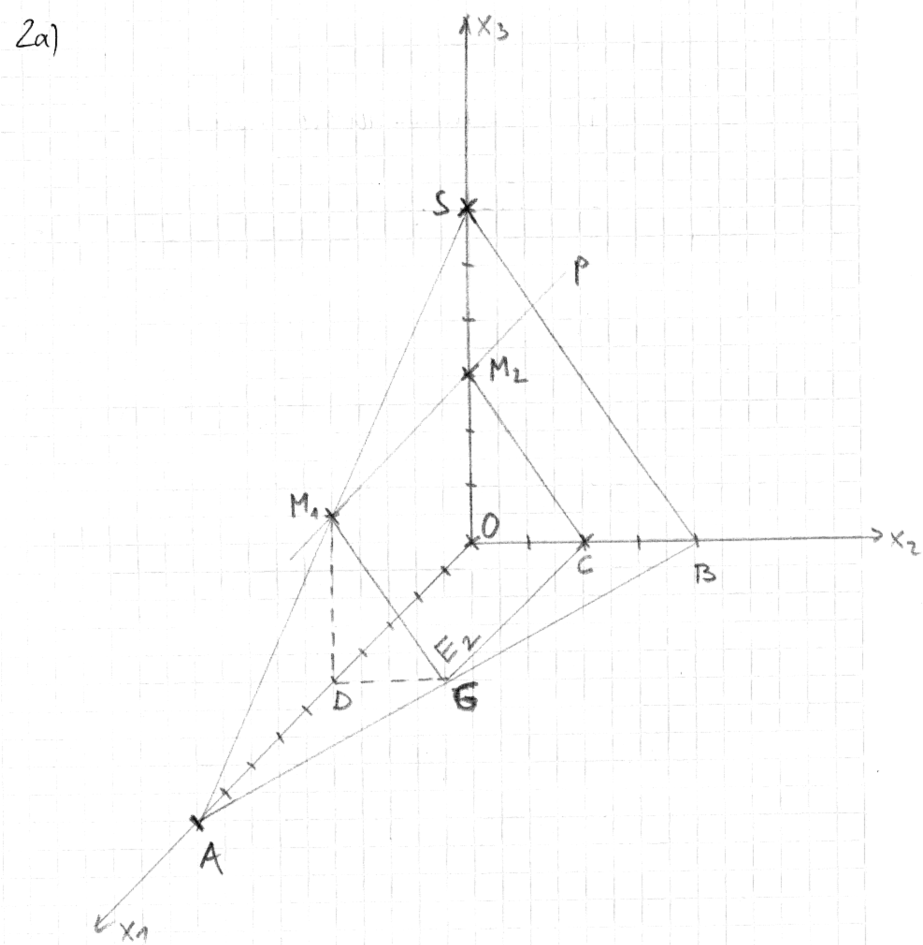
$$\left(\overrightarrow{M_1M_2} = \begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}5\\0\\3\end{pmatrix} = \begin{pmatrix}-5\\0\\0\end{pmatrix}\right)$$ nicht nötig

$M_1 \in E_t: \quad 3\cdot 0 + t\cdot 3 - 3t = 0$ für alle $t \in \mathbb{R}$

$M_2 \in E_t: \quad 3\cdot 0 + t\cdot 3 - 3t = 0$ " " "

$\Rightarrow M_1M_2 \subset E_t$ für alle $t \in \mathbb{R}$

2a)



2b) $V_p = \frac{1}{6} \left| \det(\vec{A}; \vec{B}; \vec{S}) \right|$

$= \frac{1}{6} \left| \begin{vmatrix} 10 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{vmatrix} \right|$

$= \frac{1}{6} \left| 10 \cdot 4 \cdot 6 \right| = \frac{1}{6} \, 240 = 40$

2c) $V_{AGDM_1OCM_2} = \underbrace{V_{AGDM_1}}_{\text{dreiseitige Pyramide}} + \underbrace{V_{DGM_1OCM_2}}_{\text{dreiseitiges Prisma}}$

$= \frac{1}{3} A_{DGM_1} \cdot \underbrace{\overline{AD}}_{\text{Höhe!}} + A_{DGM_1} \cdot \underbrace{\overline{OD}}_{\text{Höhe}}$

$= \frac{4}{3} A_{DGM_1} \cdot 5 \quad (\overline{AD} = \overline{OD} = \frac{1}{2} |\vec{A}|)$

$= \frac{4}{3} \cdot \frac{1}{2} |\vec{OC}| \cdot |\vec{OM_2}| \cdot 5 = \frac{4}{3} \cdot \frac{1}{2} \frac{4}{2} \cdot \frac{6}{2} \cdot 5 \quad (\vec{OC} \perp \vec{OM_2})$

$= 20 = \frac{1}{2} V_p$

3a) Abstand von M zur Ebene $\underbrace{ABC}_{F}$:

$\vec{n}^* = \begin{pmatrix} 6 \\ 15 \\ 10 \end{pmatrix}$ (Normalenvektor von F)

$|\vec{n}^*| = \sqrt{36 + 225 + 100} = \sqrt{361} = 19$

$F_{HNF}: \frac{1}{19}(6x_1 + 15x_2 + 10x_3 - 60) = 0$

$d(M;F) = \left|\frac{1}{19}(6 \cdot 1{,}2 + 15 \cdot 1{,}2 + 10 \cdot 1{,}2 - 60)\right|$

$= \left|\frac{1}{19}(7{,}2 + 18 + 12 - 60)\right|$

$= \left|\frac{1}{19}(-22{,}8)\right| = 1{,}2$

$d(M;F) = d(M;\, x_1\text{-}x_2\text{-Ebene}) = d(M;\, x_2\text{-}x_3\text{-Ebene})$

$= d(M;\, x_1\text{-}x_3\text{-Ebene}) = 1{,}2$

3b) $\vec{n}_t = \begin{pmatrix} 0 \\ 3 \\ t \end{pmatrix}$ von E_t

$|\vec{n}_t| = \sqrt{9 + t^2}$

$E_{t_{HNF}}: \frac{1}{\sqrt{9+t^2}}(3x_2 + tx_3 - 3t) = 0$

$d(M;E_t) = \frac{1}{\sqrt{9+t^2}}(3 \cdot 1{,}2 + t \cdot 1{,}2 - 3t) = 1{,}2$

$$\begin{aligned}
3{,}6 - 1{,}8t &= 1{,}2\sqrt{9+t^2} \quad |:0{,}6 \\
6 - 3t &= 2\sqrt{9+t^2} \\
(6-3t)^2 &= 4(9+t^2) \\
36 - 36t + 9t^2 &= 36 + 4t^2 \\
5t^2 - 36t &= 0 \\
t(5t - 36) &= 0
\end{aligned}$$

$\Rightarrow t = 0$ oder $t = \frac{36}{5} = 7{,}2$