

Lösungen zu den Aufgaben zum Logarithmus

1. a) $3^{2x-1} + 3^{2x+1} = 10 \Leftrightarrow 3^{2x} \cdot 3^{-1} + 3^{2x} \cdot 3^1 = 10 \quad | \cdot 3 \quad \Leftrightarrow 3^{2x} + 9 \cdot 3^{2x} = 30$
 $\Leftrightarrow 10 \cdot 3^{2x} = 30 \Leftrightarrow 3^{2x} = 3^1 \Leftrightarrow 2x = 1 \Leftrightarrow x = \frac{1}{2} \quad \Rightarrow \quad \mathbb{L} = \left\{ \frac{1}{2} \right\}$
- b) $3^{2x-1} + 3^{2x+1} = 20 \Leftrightarrow 3^{2x} \cdot 3^{-1} + 3^{2x} \cdot 3^1 = 20 \quad | \cdot 3 \quad \Leftrightarrow 3^{2x} + 9 \cdot 3^{2x} = 60$
 $\Leftrightarrow 10 \cdot 3^{2x} = 60 \Leftrightarrow 3^{2x} = 6 \Leftrightarrow 2x = \log_3 6 \Leftrightarrow x = \frac{1}{2} \cdot \log_3 6 \quad \Rightarrow \quad \mathbb{L} = \left\{ \frac{1}{2} \cdot \log_3 6 \right\}$
2. a) $\log_2 16^{-3} = \log_2 (2^4)^{-3} = \log_2 2^{-12} = -12 \cdot \log_2 2 = -12$
- b) $\log_{\frac{1}{3}} 729 = \log_{\frac{1}{3}} 3^6 = \log_{\frac{1}{3}} \left(\frac{1}{3} \right)^{-6} = -6$
- c) $\log_{0,5} 4^3 = \log_{0,5} (2^2)^3 = \log_{0,5} 2^6 = \log_{0,5} \frac{1}{2}^{-6} = -6$
3. a) $\log[(x+2)^2 - 4 + x^2] = \log[(x+2)^2 + x^2 - 4] = \log[(x+2)^2 + (x+2)(x-2)]$
 $= \log[(x+2) \cdot (x+2+x-2)] = \log[(x+2) \cdot 2x] = \log(x+2) + \log 2 + \log x$
- b) $\log \frac{4-x^2}{\sqrt[3]{y^2-6y+9}} = \log \frac{(2-x)(2+x)}{(y-3)^{\frac{2}{3}}} = \log(2-x) + \log(2+x) - \frac{2}{3} \cdot \log(y-3)$
- c) $\log \frac{ab^2}{x^2y} = \log a + 2 \log b - 2 \log x - \log y$
- d) $\log(x\sqrt{a^2-x^2}) = \log\left(x(a-x)^{\frac{1}{2}}(a+x)^{\frac{1}{2}}\right) = \log x + \frac{1}{2} \cdot \log(a-x) + \frac{1}{2} \cdot \log(a+x)$
4. $\frac{1}{2} \log a - \frac{1}{4} \log b^2 + \frac{1}{2} \log ab = \log a^{\frac{1}{2}} - \log b^{2 \cdot \frac{1}{4}} + \log(ab)^{\frac{1}{2}}$
 $= \log \sqrt{a} - \log \sqrt{b} + \log \sqrt{ab} = \log \frac{\sqrt{a} \cdot \sqrt{ab}}{\sqrt{b}} = \log a$

Lu