Lösungsblatt zu den
Übungen zu den binomischen Formeln und zum Faktorisieren

1. a) $(5x + 3y)^2 = 25x^2 + 30xy + 9y^2$

   b) $(7x^2 + 3)^2 = 49x^4 + 42x^2 + 9$

   c) $(9x – 5y)^2 = 81x^2 – 90xy + 25y^2$

   d) $(5k^2 – 12m^2)^2 = 25k^4 – 120k^2m^2 + 144m^4$

   e) $(4p^2 + 9q^2)^2 = 16p^4 + 72p^2q^2 + 81q^4$

   f) $(1{,}2x^2 – 0{,}18y^3)^2 = 1{,}44x^4 – 0{,}432x^2y^3 + 0{,}0324y^6$

   g) $(1 – 4y)(1 + 4y) = 1 – 16y^2$

   h) $(5{,}5x^2 + 0{,}75y^2)(5{,}5x^2 – 0{,}75y^2)$
   $= 30{,}25x^4 – 0{,}5625y^4$

   i) $(2 – z)^2 – (2 – z)(2 + z) + (2 + z)^2 = 4 – 4z + z^2 – (4 – z^2) + 4 + 4z + z^2 = 4 + 3z^2$

   j) $(4a + 3b^2)^2 + 3(4a + 5b^2)(4a – 5b^2) – 4(3a – 4b^2)^2$
   $= 16a^2 + 24ab^2 + 9b^4 + 3(16a^2 – 25b^4) – 4(9a^2 – 24ab^2 + 16b^4)$
   $= 16a^2 + 24ab^2 + 9b^4 + 48a^2 – 75b^4 – 36a^2 + 96ab^2 – 64b^4 = 28a^2 + 120ab^2 – 130b^4$

   k) $(x + y)^2 + (x – y)^2$
   $= x^2 + 2xy + y^2 + x^2 – 2xy + y^2 = 2x^2 + 2y^2$

   l) $(x + y)^2 – (x – y)^2$
   $= x^2 + 2xy + y^2 – (x^2 – 2xy + y^2) = 4xy$

   m) $(5a + 7b)^2 – (5a – 7b)^2 + (5a – 7b)(5a + 7b)$
   $= 25a^2 + 70ab + 49b^2 – (25a^2 – 70ab + 49b^2) + 25a^2 – 49b^2 = 25a^2 + 140ab – 49b^2$

   n) $(x + y + z)(x + y – z) = [(x + y) + z][(x + y) – z] = (x + y)^2 – z^2 = x^2 + 2xy + y^2 – z^2$

   o) $(4a – 7b – 3c)(4a – 7b + 3c) = [(4a – 7b) – 3c][(4a – 7b) + 3c] = (4a – 7b)^2 – 9c^2 = 16a^2 – 56ab + 49b^2 – 9c^2$

   p) $(x + y – z)^2 = [(x + y) – z]^2 = (x + y)^2 – 2\cdot(x + y)\cdot z + z^2 = x^2 + 2xy + y^2 – 2xz – 2yz + z^2$

   q) $(1{,}3a – 1{,}8b + 1{,}7c)^2 = [(1{,}3a – 1{,}8b) + 1{,}7c]^2 = (1{,}3a – 1{,}8b)^2 + 2\cdot(1{,}3a – 1{,}8b)\cdot 1{,}7c + 2{,}89c^2$
   $= 1{,}69a^2 – 4{,}68ab + 3{,}24b^2 + 4{,}42ac – 6{,}12bc + 2{,}89c^2$

   r) $[(1 – x)^2 + (1 + x)^2](1 + x)(1 – x) – [(x – 1)^2 + (x + 1)^2](x + 1)(x – 1)$
   $= [1 – 2x + x^2 + 1 + 2x + x^2](1 – x^2) – [x^2 – 2x + 1 + x^2 + 2x + 1](x^2 – 1)$
   $= (2 + 2x^2)(1 – x^2) – (2x^2 + 2)(x^2 – 1) = 2\cdot(1 + x^2)(1 – x^2) – 2\cdot(x^2 + 1)(x^2 – 1) = 2\cdot(1 – x^4) – 2\cdot(x^4 – 1)$
   $= 2 – 2x^4 – 2x^4 + 2 = 4 – 4x^4$

2. a) $25x^2 – 70xy + 49y^2 = (5x – 7y)^2$

   b) $4a^2 – 2ab + 0{,}25b^2 = (2a – 0{,}5b)^2$

   c) $(4x)^2 – 8xy + y^2 = (4x – y)^2$

   d) $196a^2 – 49y^2 = (14a – 7y)(14a + 7y)$

   e) $289x^4y^2 – 361x^4y^2 = –72x^4y^2$

   f) $289x^4y^2 – 361x^2y^4 = x^2y^2\cdot(289x^2 – 361y^2) = x^2y^2\cdot(17x – 19y)(17x + 19y)$

   g) $289x^4y^2 + 361x^4y^2 = 650x^4y^2$

   h) $4x^2 + 0{,}8xy + 0{,}04y^2 = (2x + 0{,}2y)^2$

   i) $xy + 2x + 3y + 6 = x\cdot(y + 2) + 3\cdot(y + 2) = (y + 2)(x + 3)$

   j) $xy + 20 + 4x + 5y = xy + 4x + 5y + 20$
   $= x\cdot(y + 4) + 5\cdot(y + 4) = (y + 4)(x + 5)$

   k) $x^3 + x^2 + x + 1 = x^2\cdot(x + 1) + 1\cdot(x + 1) = (x + 1)(x^2 + 1)$

   l) $cy – dy + 5xy = y\cdot(c – d + 5x)$

   m) $x^2 + 2x – 8 = (x + 4)(x – 2)$

   n) $x^2 – 3x – 4 = (x – 4)(x + 1)$

   o) $x^2 – 14x + 40 = (x – 10)(x – 4)$

   p) $18 – 11x + x^2 = (2 – x)(9 – x)$

   q) $y^2 – 23x + 42x^2 = (y – 21y)(x – 2y)$

   r) $25x^2 – 16 = (5x – 4)(5x + 4)$

   s) $x^2 – 11$ nicht möglich

   t) $9x^2 – 144 = (3x – 12)(3x + 12)$

   u) $4x^2 + 9$ nicht möglich

   v) $x^4 – y^4 = (x^2 – y^2)(x^2 + y^2) = (x – y)(x + y)(x^2 + y^2)$

   w) $x^8 – y^6 = (x^4 – y^3)(x^4 + y^3)$

   x) $24x^2y + 8x^2y + 18xy^2 = 32x^2y + 18xy^2$
   $= 2xy\cdot(16x + 9y)$

   y) $0{,}27x^2y – 507y^3 = 3y\cdot(0{,}09x^2 – 169y^2) = 3y\cdot(0{,}3x – 13y)(0{,}3x + 13y)$

   z) $2x^3y + 128xy^3 – 32x^2y^2 = 2xy\cdot(x^2 + 64y^2 – 16xy) = 2xy\cdot(x^2 – 16xy + 64y^2) = 2xy\cdot(x – 8)^2$

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