Aufgaben zu den Potenzgesetzen

1. a) $(u^m + u^3)(u^{4-n} + u)$ b) $(v^p + v^q)(v^{p+2} - v^{q+2})$
   c) $(5x^{3k+2l} + 7x^{2k-7l} - 11x^{2k-l}) \cdot 3x^{2k-l}$ d) $(x^{3n+3} + x^{2n+2} + x^{n+1} + 1)(x^{n+1} - 1)$
   e) $(z^{3m-n} - z^{2m} + z^{m+n} - z^{2n} + z^{3n-m})(z^m + z^n)$

2. a) $(\frac{3}{4}x^k y^{l+1} - \frac{1}{8}x^{3k} y^{4l-7} + \frac{1}{2}x^{5k} y^{2l+4}) : \frac{1}{8}x^k y^l$
   b) $(2\frac{1}{2}a^{7m+5n} - 3\frac{1}{3}a^{6m-2n} + \frac{5}{6}a^{5m+2n}) : \frac{5}{6}a^{5m-3n}$
   c) $(10u^{8p+12q} + 4u^{7p-4q}) : 2u^{3p+5q} - (9u^{9p-2q} - 18u^{8p-18q}) : 3u^{4p-9q}$

3. Fasse zusammen:
   a) $5 \cdot 3^k - 11 \cdot 3^k + 4 \cdot 3^k$ b) $7 \cdot 3^k + 3^{k+1}$
   c) $6 \cdot 2^n - 11 \cdot 2^{n-1}$ d) $6 \cdot 5^{k+1} - 14 \cdot 5^k - 80 \cdot 5^{k-1}$
   e) $s^3 t^{l-1} - t^{l+2}$ f) $3a(x-y)^p - 2b(x-y)^p$
   g) $a^2(a^2+1)^{m-1} + (a^2+1)^{m-1}$ h) $(u^3-1)^k - u^3(u^3-1)^k$

4. Zerlege so weit wie möglich in Faktoren:
   a) $x^4 y^2 - x^2 y^4$ b) $4x^7 - x^5$ c) $2p^8 - 2p^4 q^4$ d) $a^6 - b^6$
   e) $y^m - y^{m+2}$ f) $c^{k+1} - c^{k-1}$ g) $e^{n-2} - e^{n-1}$ h) $r^{l-1} + r^{l+2}$
   i) $w^{2n} - 25$ k) $x^{2m} - y^{2m}$ l) $b^{2k} - c^{2l}$ m) $u^{2p+1} - u^{2q+1}$
   n) $a^{m+2} + 4a^{m+1} + 4a^m$ o) $9y^{l+2} - 12y^l + 4y^{l-2}$
   p) $x^{2n} + 2x^n z^l + z^{2l}$ q) $25v^{2k} - 20v^{k+m} + 4v^{2m}$

5. Kürze so weit wie möglich:
   a) $\dfrac{x^5 - x^4}{x^5 - x^3}$ b) $\dfrac{a^k - a^{k+1}}{a^k - a^{k-1}}$ e) $\dfrac{x^{2m} + 2x^m y^n + y^{2n}}{x^{2m} - y^{2n}}$
   d) $\dfrac{u^{l+1} - u w^l}{u^{2l} - w^{2l}}$ e) $\dfrac{z^{k-1} - 2z^k + z^{k+1}}{z^{k+1} - z^k}$ f) $\dfrac{a^{4p+1} - 2a^{2p+1} b^p + ab^{2p}}{a^{4p} b - b^{2p+1}}$

6. Fasse jeweils zu einem einzigen Bruchterm zusammen:
   a) $\dfrac{1}{x^2} - \dfrac{1}{x}$ b) $\dfrac{2}{a^6} - \dfrac{3}{a^4} + \dfrac{1}{a^3}$ c) $\dfrac{1}{z^3} + \dfrac{1-z}{z^4}$
   d) $\dfrac{1-x^2}{x^8} + \dfrac{1+x}{x^6} - \dfrac{1}{x^5}$ e) $\dfrac{b}{ay^2} - \dfrac{c}{by} + \dfrac{a}{cy^2}$
   f) $\dfrac{3z}{x^2 y} + \dfrac{z^2}{xy^3} + \dfrac{2z^3}{x^3 y^2}$ g) $\dfrac{1+c}{c^n} - \dfrac{1-c}{c^{n-1}} - \dfrac{1}{c^{n-2}}$
   h) $\dfrac{3a^2 + 1}{a^{3n+1}} - \dfrac{2a-1}{a^{3n}} - \dfrac{1}{a^{3n-1}}$ i) $\dfrac{1}{u^{k+1} v^{l-3}} - \dfrac{2}{u^k v^{l-2}} + \dfrac{1}{u^{k-1} v^{l-1}}$
   k) $\dfrac{1}{x^{l-3}} - \dfrac{x^3 + 3}{x^l} + \dfrac{3x^2 - 9}{x^{l+2}} + \dfrac{5x^{l-3} - 1}{x^{2l-1}} + \dfrac{4x^{l-2} + x + 1}{x^{2l}}$
   l) $\dfrac{a^k + b^k}{a^k - b^k} - \dfrac{a^k - b^k}{a^k + b^k}$ m) $\dfrac{x^m + x^n}{x^m - x^n} + \dfrac{x^m - x^n}{x^m + x^n} - \dfrac{x^{2m} + x^{2n}}{x^{2m} - x^{2n}}$
   n) $\dfrac{1}{c^k} - \dfrac{1}{c^l}$ o) $\dfrac{2}{z^m} + \dfrac{1}{z^3}$ p) $\dfrac{3}{a^{p+1}} - \dfrac{5}{a^{q+1}}$

7. a) $\dfrac{p^7}{r} \cdot \left(\dfrac{q^5}{p^4} : \dfrac{q^8}{r^4}\right)$ b) $\left(\dfrac{6ay^3}{5bx^4} \cdot \dfrac{2a^3 x^5}{3b^2 y^4}\right) : \dfrac{a^2 x^2}{by}$
   c) $\dfrac{v^{2n}}{w^{n-1}} : \left(\dfrac{u^2}{w^{n+1}} \cdot \dfrac{v^{3n}}{u^{n+1}}\right)$ d) $\left(\dfrac{p^{3n+2}}{q^{m-1}} : \dfrac{q^3}{s^n}\right) : \left(\dfrac{p^{2n+2}}{s^{n-1}} : \dfrac{q^2}{s^{2n-3}}\right)$

8. a) $\left(\dfrac{5a^p}{9b^q}\right)^{pq}$ b) $\left(\dfrac{1}{u^{k-1} v^{k+1}}\right)^{k-1}$ c) $\left(-\dfrac{7a^m x^2}{12b^l y^3}\right)^{4k-3}$
   d) $\left[\left(\dfrac{5bx^2}{2ay^2}\right)^2 \cdot \left(\dfrac{3xy^2}{5ab^2}\right)^3\right] : \left(\dfrac{3x^4 y}{2a^2 b^3}\right)^2$ e) $\left(\dfrac{2p^n q^2}{3vw^{2n-1}}\right)^3 : \left(\dfrac{4p^{n-1} q^3}{9v^2 w^{3n-2}}\right)^2$

9. Verwandle in Terme, in denen als Exponenten nur Variable auftreten:
   a) 3^{2n} b) 5^{m+1} c) 6^{n-2} d) 4^{2k+3} e) 10^{3k-5}

10. Kürze:
    a) $\dfrac{6^5 \cdot 35^4}{9 \cdot 14^4 \cdot 15^3}$ b) $\dfrac{24^4 \cdot 40^3 \cdot 45^2}{16^2 \cdot 36^4 \cdot 75^3}$ c) $\dfrac{4^n \cdot 25^{n+1}}{10^{2n+1}}$
    d) $\dfrac{132^6 \cdot 175^6}{5^{12} \cdot 21^6 \cdot 44^6}$ e) $\dfrac{(12^3 \cdot 15 \cdot 25^2)^2}{(5^2 \cdot 6^3 \cdot 10)^3}$ f) $\dfrac{(6^{n-1} \cdot 14^n \cdot 21^2)^3}{(27 \cdot 28^3)^{n+1}}$

11. a) $\dfrac{2(6a^3 b^4 x^2)^3}{5(4a^2 b^3 x)^4}$ b) $\dfrac{(8u^3 w^2)^{n-1}}{(4u^2 w)^{2n+1}}$ c) $\dfrac{(a^4 b^2 x y^3)^{2n+1}}{(a^{n+1} b^{n-1} x^{2n+1} y)^4}$